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$$\therefore\ d_a=\frac{2bc}{b+c}\cos\tfrac{1}{2}A=\frac{2\sqrt{[s(s-a)bc]}}{b+c},\text{ where } 2s=a+b+c.$$

$$\text{Hence, if } d_a=d_b,\ \frac{2\sqrt{[s(s-a)bc]}}{b+c}=\frac{2\sqrt{[s(s-b)ac]}}{a+c}.$$

$$\therefore\ 2(a+c)^2(s-a)b=2(b+c)^2(s-b)a.$$

$$\therefore\ (a+c)^2(b+c-a)b-(b+c)^2(a+c-b)a=0.$$

$$\therefore\ (a-b)[c^3+c^2(a+b)+3abc+ab(a+b)]=0.$$

Since the second factor is positive, it does not vanish. Hence $a=b$.

We have finally received an elementary direct proof of this theorem. The proof by Professor Hopkins is, I believe, without a flaw, and is the proof so long sought for by a number of mathematicians, among whom was Isaac Todhunter. This demonstration of Professor Hopkins' was examined by one of the ablest mathematicians in this country, and was pronounced by him to be correct. The trigonometric proof by Dr. Dickson is also flawless. We are glad to publish both of these proofs since the demonstration which we gave in Vol. VII, page 223, has been assailed.

Last January a year ago, we received an indirect proof from Prof. A. Anderson, of the University of North Carolina. Professor Anderson's demonstration is free from error and is substantially the proof given in Dr. Halsted's Synthetic Geometry, page 44, though Professor Anderson's figure is drawn quite differently from the one in Dr. Halsted's Geometry.

We shall be pleased to receive, from our readers, opinions on the above demonstrations.

153. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

If P, P', Q, Q' be the extremities of two chords of a conic section, and both chords pass through the point A, show that the sum of the squares of the reciprocals of AP, AP', AQ, AQ' is constant.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

$$\frac{1}{AP}=\frac{1+e\cos\theta}{l},\quad \frac{1}{AP'}=\frac{1+e\cos(\pi+\theta)}{l}=\frac{1-e\cos\theta}{l}.$$

$$\frac{1}{AQ}=\frac{1+e\cos(\tfrac{1}{2}\pi+\theta)}{l}=\frac{1-e\sin\theta}{l}.$$

$$\frac{1}{AQ'}=\frac{1+e\cos(3\pi/2+\theta)}{l}=\frac{1+e\sin\theta}{l}.$$

$$\therefore\frac{1}{(AP)^2}+\frac{1}{(AP')^2}+\frac{1}{(AQ)^2}+\frac{1}{(AQ')^2}$$

$$=\frac{(1+e\cos\theta)^2+(1-e\cos\theta)^2+(1-e\sin\theta)^2+(1+e\sin\theta)^2}{l^2}=\frac{2(2+e^2)}{l^2}=\text{a constant}.$$

158. Proposed by JOHN MACNIE, A. M., Professor of Latin, University of North Dakota.

Show by a simple diagram that:

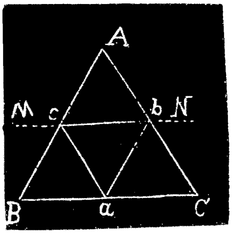
(a) If the angle-sum of an equilateral triangle is constant. that constant is a straight angle.

(*b*) If the angle-sum is less than a straight angle, the sum increases as the triangle grows less.

(*c*) If the angle-sum is greater than a straight angle, the sum decreases as the triangle grows less.

Solution by the PROPOSER.

Join the mid-points a, b, c, of the sides of an equilateral triangle ABC. The triangles with vertices A, B, C, being equal, we have $ab=bc=ac$, and $\angle Bac=\angle Cab=\angle Abc$, etc.



I. If $\angle A=\angle bac$, then $\angle A+\angle Abc+\angle Acb=\angle bac+Bac+\angle Cab=180°$, and it is easily shown that $\triangle abc=\triangle Abc$, etc.

Lemma to II and III.

If MN be a line moving towards coincidence with BC, so as always to cut off equal parts on AB, AC; then, according as the angle-sum of ABC is $<$ *or* $>$ *180°, so will that of* $\triangle Abc$ *be* $<$ *or* $>$ *180°.*

For otherwise, as the angle-sum of Abc would vary from $>$ or $<$ 180° to $<$ or $>$ 180°, there would be some position of MN in which the angle-sum of Abc would be 180°, with consequences incompatible with the hypothesis.

II. If $\angle A<60°$, then $\angle A+\angle Abc+\angle Acb<180°<\angle abc+\angle Abc+\angle Cba$. $\therefore \angle abc>\angle A$.

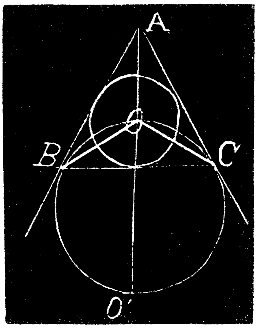
III. If $\angle A>60°$, then $\angle A+\angle Abc+\angle Acb>180°>\angle abc+\angle Abc+\angle Cba$. $\therefore \angle abc<\angle A$.

162. Proposed by J. D. PALMER, Providence, Ky.

Given the distances from the vertices of a triangle, ABC, to the center of the in-circle, to construct the triangle.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

AO, BO, $CO=a$, b, c, respectively, where O is the center of the in-circle; BC, AC, $AB=x$, y, z, respectively. Let O_1 be the center of the ex-circle opposite A. Then



$$AO^2=\frac{(p-x)^2}{\cos^2\frac{1}{2}A},\text{ where } p=\tfrac{1}{2}(x+y+z).$$

$$\therefore AO^2=\left(\frac{p-x}{p}\right)yz=yz-\frac{xyz}{p}=yz-4Rr=a^2.$$

Similarly, $$BO^2=\left(\frac{p-y}{p}\right)xz=xz-4Rr=b^2.$$

$$CO^2=\left(\frac{p-z}{p}\right)xy=xy-4Rr=c^2.$$